12.3

What *Factored* Into It?

Factoring Polynomials

LEARNING GOALS

In this lesson, you will:

- Factor polynomials by determining the greatest common factor.
- Factor polynomials by using multiplication tables.

The history of the word "multiply" is an interesting one. You probably know that "multi-" means "many." But did you know that the rest of the word, based on the Latin "plicare," means "to fold"?

This is why you might read in older texts that someone increased their money "twofold" or "tenfold."

Multiplication is closely related to folding. Can you see how?

PROBLEM 1 What About the Other Way Around?

In the previous lesson, you multiplied polynomials. More specifically, you multiplied two linear expressions to determine a quadratic expression. In this lesson, you will go in reverse and think about how to take a polynomial represented as the sum of terms and write an equivalent expression in factored form, if it is possible. To factor an expression means to rewrite the expression as a product of factors.

One way to factor an expression is to factor out the greatest common factor first.

Consider the polynomial $3x + 15$.

The greatest common factor is 3.

$$3x + 15 = 3x + 3(5)$$
$$= 3(x + 5)$$

Therefore, $3x + 15 = 3(x + 5)$.

In order to factor out a greatest common factor, use the Distributive Property in reverse. Recall that, using the Distributive Property, you can rewrite the expression $a(b + c)$ as $ab + ac$.

1. Factor out the greatest common factor for each polynomial, if possible.

a. $4x + 12$

b. $x^3 - 5x$

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c. $3x^2 - 9x - 3$

d. $-x - 7$

e. $2x - 11$

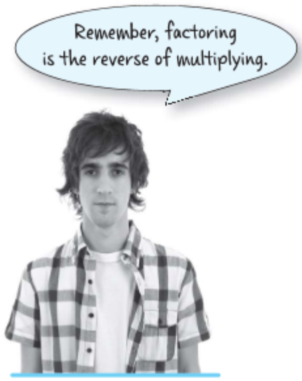
f. $5x^2 - 10x + 5$

2. How can you check to see if you factored out the greatest common factor of each correctly?

PROBLEM 2 Factoring Trinomials

In the previous chapter, you used a graphing calculator to rewrite a quadratic expression in factored form, $ax^2 + bx + c = a(x - r_1)(x - r_2)$.

Now, let's consider a strategy for factoring quadratic expressions without the use of technology. Understanding that the product of two linear expressions produces a quadratic expression is necessary for understanding how to factor a quadratic expression.



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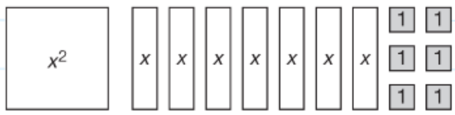
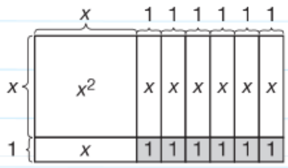
An area model can be used to factor $x^2 + 7x + 6$.

First, represent each part of the trinomial as a piece of the area model. In this problem, $x^2 + 7x + 6$ consists of one x^2 block, seven x blocks, and six constant blocks.



Second, use all of the pieces to form a rectangle. In this problem, the parts can only be arranged in one way.

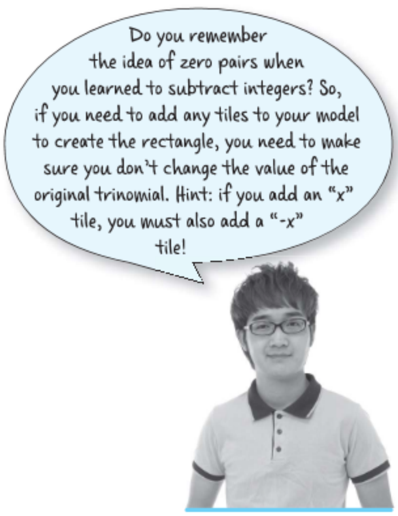


1. Write the trinomial as the product of the two factors.

   $x^2 + 7x + 6 =$

2. Factor each trinomial using an area model.

   a. $x^2 + 5x + 4 =$



b. $x^2 - 6x + 9 =$

c. $x^2 + 5x - 6 =$

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3. Look back at the quadratic trinomials you just factored using area models. What do you notice about the constant term of the trinomial, c, and the constants of the binomial factors r_1 and r_2?

Let's consider using a multiplication table model to factor trinomials.

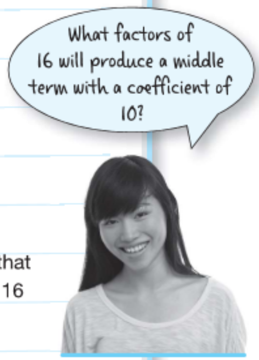
Factor the trinomial $x^2 + 10x + 16$.

Start by writing the leading term (x^2) and the constant term (16) in the table.

·		
	x^2	
		16

Factor the leading term.

·	x	
x	x^2	
		16



Experiment with factors of the constant term to determine the pair that produces the correct coefficient for the middle term. The factors of 16 are (1)(16), (2)(8), and (4)(4).

·	x	8
x	x^2	$8x$
2	$2x$	16

The sum of $2x$ and $8x$ is $10x$.

So, $x^2 + 10x + 16 = (x + 2)(x + 8)$.

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4. Explain why the other factor pairs for $c = 16$ do not work.
   - a. (1)(16)
   - b. (4)(4)

5. Use multiplication tables to factor each trinomial.
   - a. $x^2 + 9x + 20$

b. $x^2 + 11x + 18$

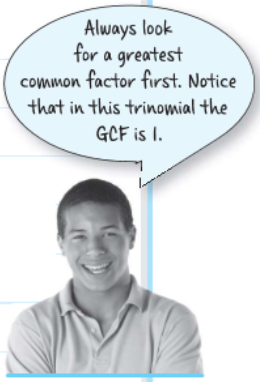
Another method for factoring a trinomial is trial and error, using the factors of the leading term, ax^2, and the constant term, c.

Factor the trinomial $x^2 - 10x - 24$.

To factor the quadratic expression, you must consider the factors of ax^2 and c.

Factors of the leading term, x^2, are x and x.

Factors of the constant term, -24, are: -1, 24 and 1, -24
-2, 12 and 2, -12
-3, 8 and 3, -8
-4, 6 and 4, -6.



Use these factor pairs to write binomial products. Determine which binomials produce the correct middle term.

$(x - 1)(x + 24) = x^2 + 23x - 24$ not correct
$(x + 1)(x - 24) = x^2 - 23x - 24$ not correct

$(x - 2)(x + 12) = x^2 + 10x - 24$ not correct
$(x + 2)(x - 12) = x^2 - 10x - 24$ correct

$(x - 3)(x + 8) = x^2 + 5x - 24$ not correct
$(x + 3)(x - 8) = x^2 - 5x - 24$ not correct

$(x - 4)(x + 6) = x^2 + 2x - 24$ not correct
$(x + 4)(x - 6) = x^2 - 2x - 24$ not correct

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6. Factor each trinomial using the method from the worked example. List the factor pairs.

   a. $x^2 + 5x - 24$

   b. $x^2 - 3x - 28$

7. Consider the two examples shown.

$2x^2 - 3x - 5$

$\cdot$	x	1
$2x$	$2x^2$	$2x$
-5	$-5x$	-5

$2x^2 - 3x - 5 = (2x - 5)(x + 1)$

$2x^2 + 3x - 5$

$\cdot$	x	-1
$2x$	x^2	$-2x$
5	$5x$	-5

$2x^2 + 3x - 5 = (2x + 5)(x - 1)$

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   a. Compare the two given trinomials. What is the same and what is different about the a, b, and c values?

   b. Compare the factored form of each trinomial. What do you notice?

8. Choose from the list to write the correct factored form for each trinomial.

a. $x^2 + 5x + 4 =$ ______________

$x^2 - 5x + 4 =$ ______________

$x^2 + 3x - 4 =$ ______________

$x^2 - 3x - 4 =$ ______________

- $(x + 1)(x - 4)$
- $(x + 1)(x + 4)$
- $(x - 1)(x + 4)$
- $(x - 1)(x - 4)$

b. $2x^2 + 7x + 3 =$ ______________

$2x^2 - 7x + 3 =$ ______________

$2x^2 - 5x - 3 =$ ______________

$2x^2 + 5x - 3 =$ ______________

- $(2x - 1)(x - 3)$
- $(2x - 1)(x + 3)$
- $(2x + 1)(x + 3)$
- $(2x + 1)(x - 3)$

c. $x^2 + 7x + 10 =$ ______________

$x^2 - 7x + 10 =$ ______________

$x^2 - 3x - 10 =$ ______________

$x^2 + 3x - 10 =$ ______________

- $(x - 2)(x + 5)$
- $(x + 2)(x + 5)$
- $(x - 2)(x - 5)$
- $(x + 2)(x - 5)$

9. Analyze the signs of each quadratic expression written in standard form and the operations in the binomial factors in Question 8. Then complete each sentence.

the same	both positive	one positive and one negative
different	both negative	

a. If the constant term is positive, then the operations in the binomial factors are ______________.

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b. If the constant term is positive and the middle term is positive, then the operations in the binomial factors are ______________.

c. If the constant term is positive and the middle term is negative, then the operations in the binomial factors are ______________.

d. If the constant term is negative, then the operations in the binomial factors are ______________.

e. If the constant term is negative and the middle term is positive, then the operations in the binomial factors are ______________.

f. If the constant term is negative and the middle term is negative, then the operations in the binomial factors are ______________.

10. Factor each quadratic expression.

a. $x^2 + 8x + 15 =$ ______

$x^2 - 8x + 15 =$ ______

$x^2 + 2x - 15 =$ ______

$x^2 - 2x - 15 =$ ______

b. $x^2 + 10x + 24 =$ ______

$x^2 - 10x + 24 =$ ______

$x^2 + 2x - 24 =$ ______

$x^2 - 2x - 24 =$ ______

11. Elaine, Maggie, and Grace were asked to factor the trinomial $15 + 2x - x^2$.

Grace

$15 + 2x - x^2$

$(5 - x)(3 + x)$

Elaine

$15 + 2x - x^2$

$(5 - x)(3 + x)$

$(x - 5)(x + 3)$

Maggie

$15 + 2x - x^2$

$-x^2 + 2x + 15$

$-(x^2 - 2x - 15)$

$-(x - 5)(x + 3)$

Who's correct? Determine which student is correct and explain how that student determined the factored form. If a student is not correct, state why and make the correction.

12. Marilyn and Jake were working together to factor the trinomial $4x^2 + 22x + 24$. They first noticed that there was a greatest common factor and rewrote the trinomial as

$$2(2x^2 + 11x + 12).$$

Next, they considered the factor pairs for $2x^2$ and the factor pairs for 12.

$2x^2$: $(2x)\ (x)$

12: (1) (12)
(2) (6)
(3) (4)

Marilyn listed all out all the possible combinations.

$2(2x + 1)(x + 12)$
$2(2x + 12)(x + 1)$

$2(2x + 2)(x + 6)$
$2(2x + 6)(x + 2)$

$2(2x + 3)(x + 4)$
$2(2x + 4)(x + 3)$

Jake immediately eliminated four out of the six possible combinations because the terms of one of the linear expressions contained common factors.

$2(2x + 1)(x + 12)$
~~$2(2x + 12)(x + 1)$~~

~~$2(2x + 2)(x + 6)$~~
~~$2(2x + 6)(x + 2)$~~

$2(2x + 3)(x + 4)$
~~$2(2x + 4)(x + 3)$~~

Explain Jake's reasoning. Then circle the correct factored form of $4x^2 + 22x + 24$.

Talk the Talk

1. Factor each polynomial completely. First, determine if there is a greatest common factor, and then write the polynomial in factored form.

   a. $x^2 - 9x - 10$

   b. $4x^2 - 20x + 16$

   c. $-20 + 9b - b^2$

   d. $3y^2 - 8y - 3$

   e. $7x^2 - 7x - 56$

   f. $3y^3 - 27y^2 - 30y$

2. Use the word bank to complete each sentence. Then explain your reasoning.

always	sometimes	never

   a. The product of two linear expressions will ________________ be a trinomial with a degree of 3.

   b. The two binomial factors of a quadratic expression will ________________ have a degree of one.

   c. The factoring of a quadratic expression will ____________________ result in two binomials.

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Be prepared to share your solutions and methods.